Correctness proofs of distributed systems with Isabelle/HOL

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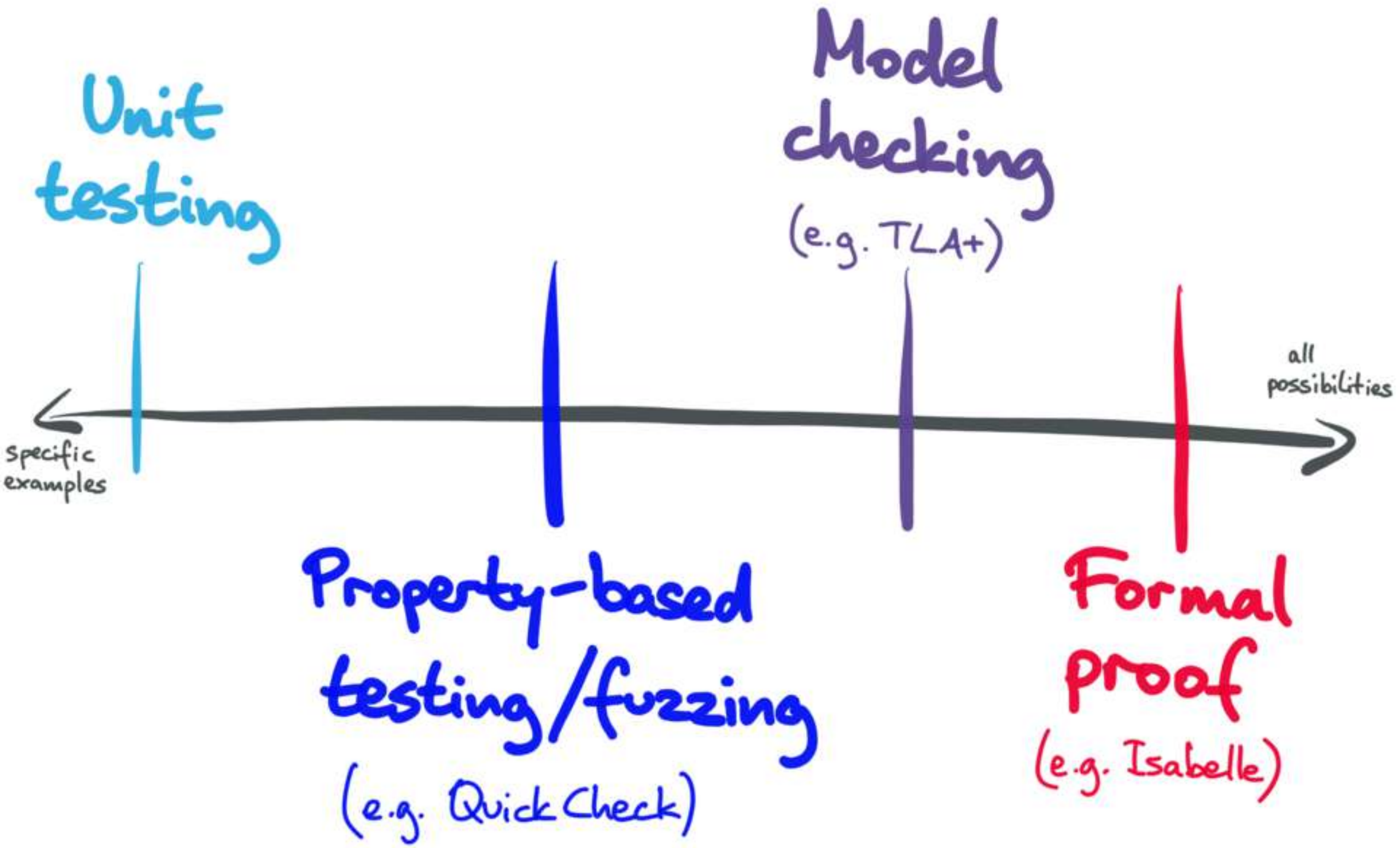
Unit testing
Property-based testing/fuzzing
(e.g. QuickCheck)
Model checking
(e.g. TLA+)
Formal proof
(e.g. Isabelle)
specific examples
all possibilities

WHY BOTHER?

- Subtle algorithms
  (correctness not obvious)
- Complicated state space
  (e.g. distributed systems, concurrency)
- For better human understanding
  (forcing yourself to be thorough & precise)

WHY BOTHER?

"Isabelle is the world's most complicated video game"

— Dominic Mulligan

DISTRIBUTED SYSTEMS

Approach: model the system using Isabelle/HOL data structures (lists, sets, ...)

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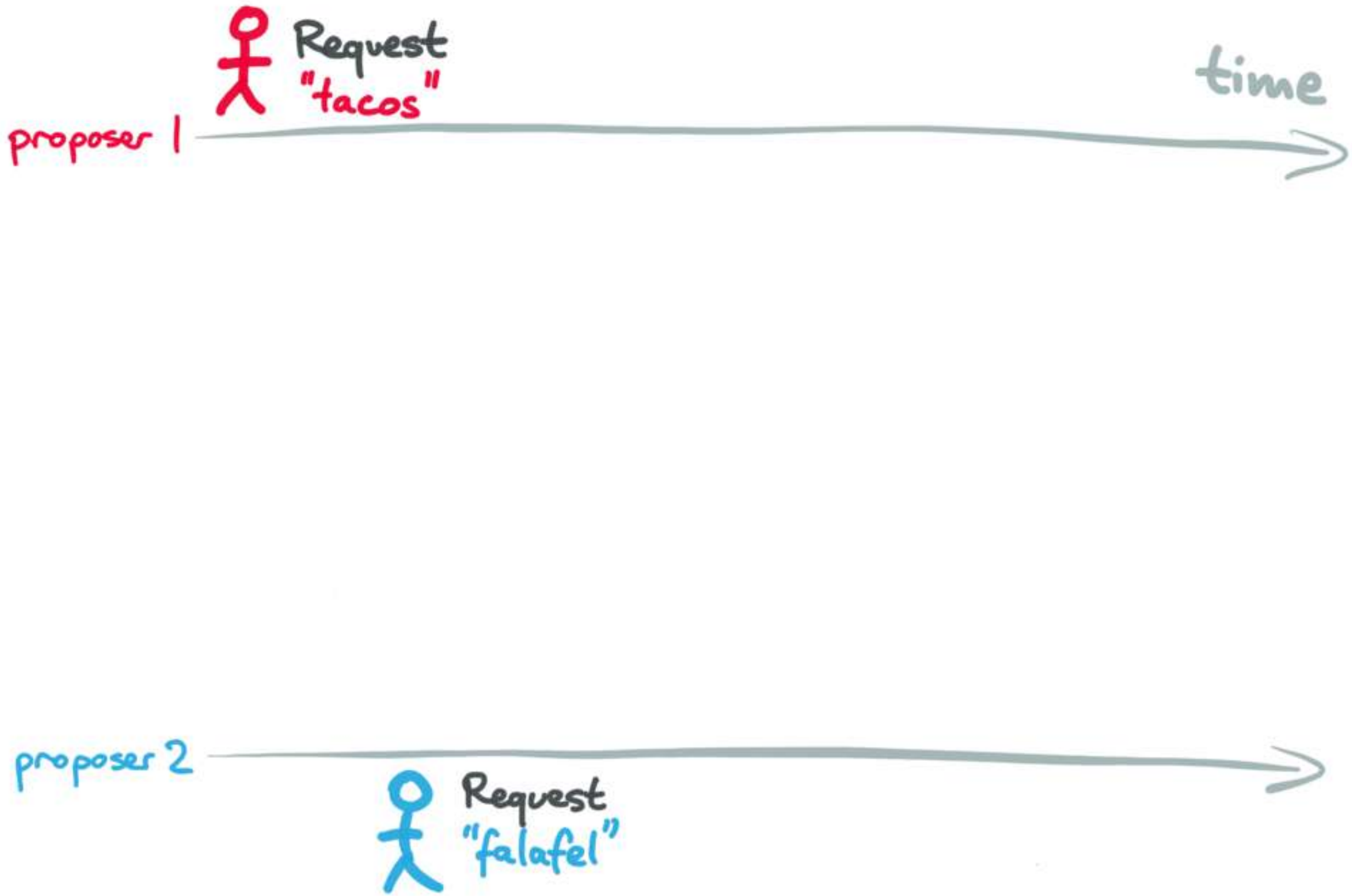
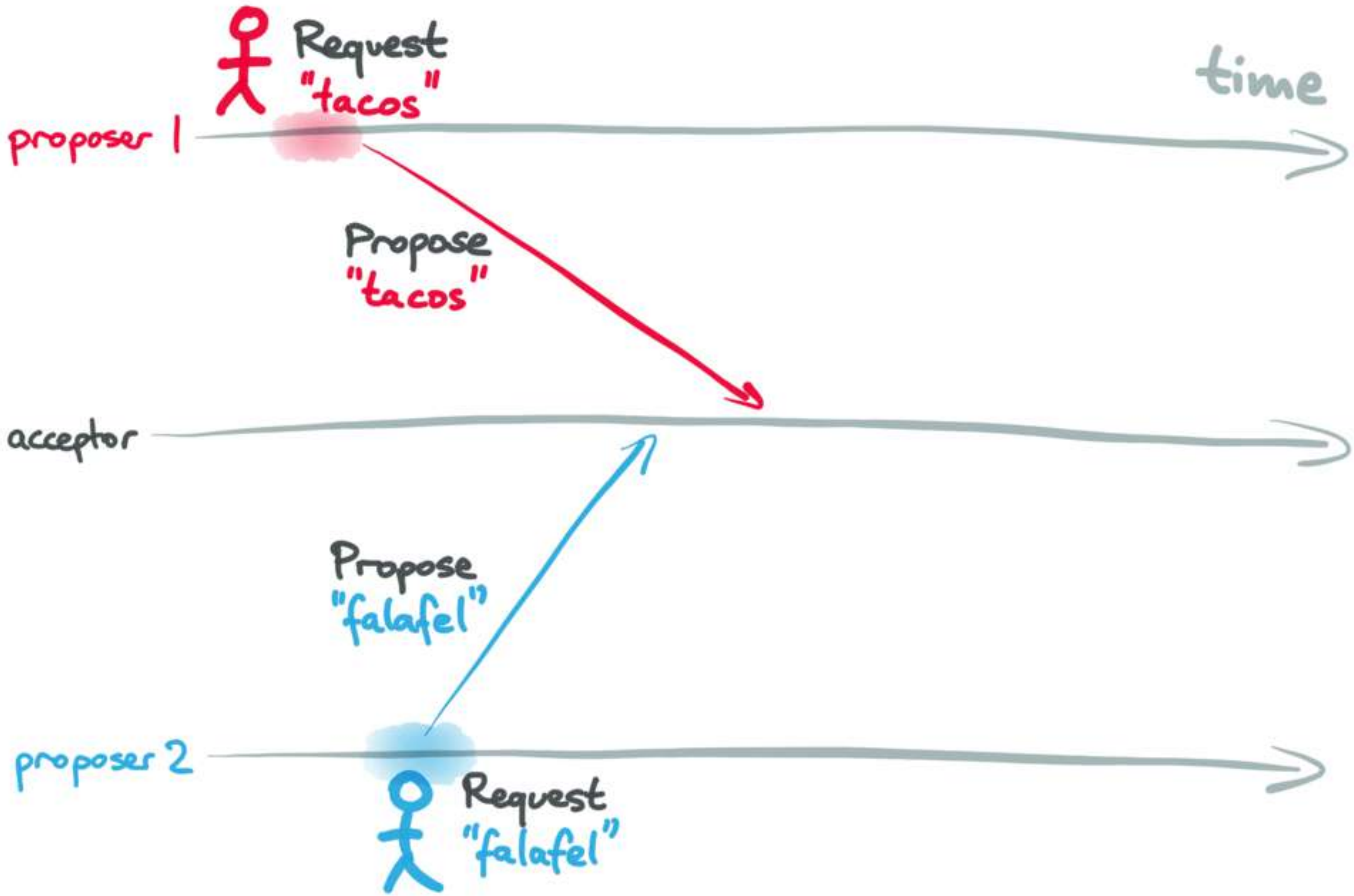
Example problem: consensus

(a simple, non-fault-tolerant algorithm – can't fit Paxos/Raft in this talk)

THE AGREEMENT PROPERTY

If any two processes learn decided values, those values are the same.


Request
"tacos"
time
proposer 1
proposer 2
Request
"falafel"


Request
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proposer 1
Propose
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Propose
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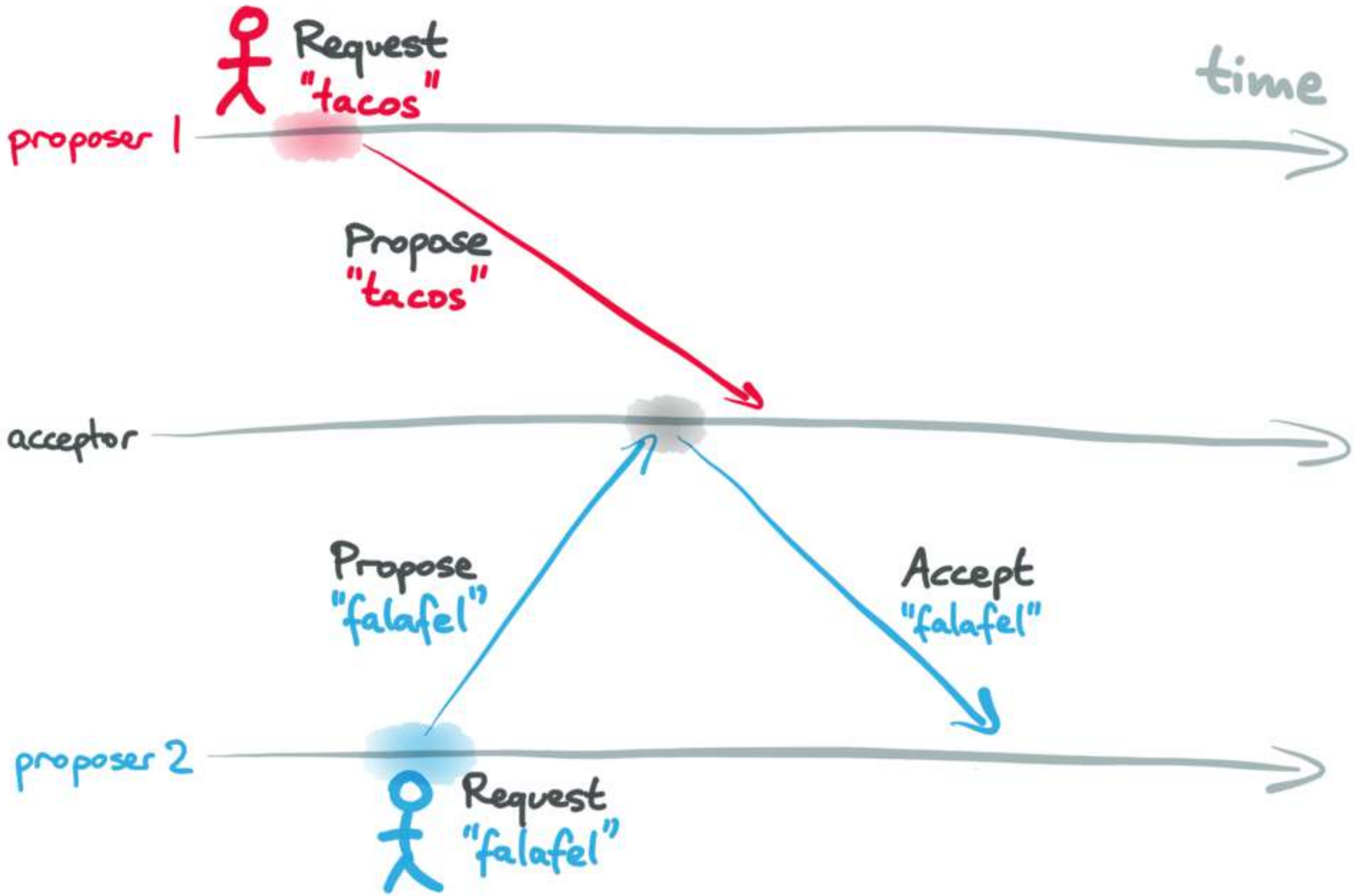

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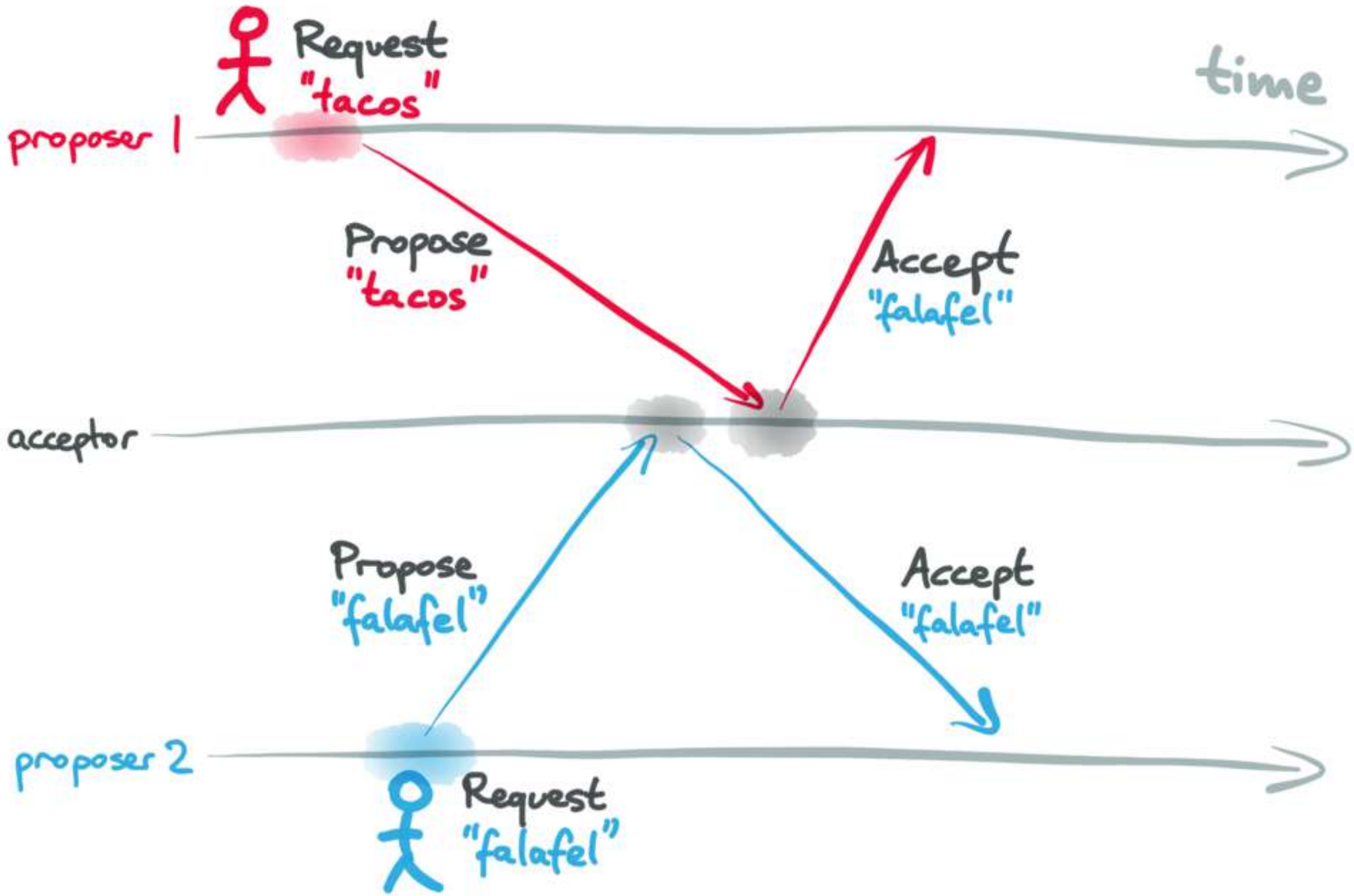
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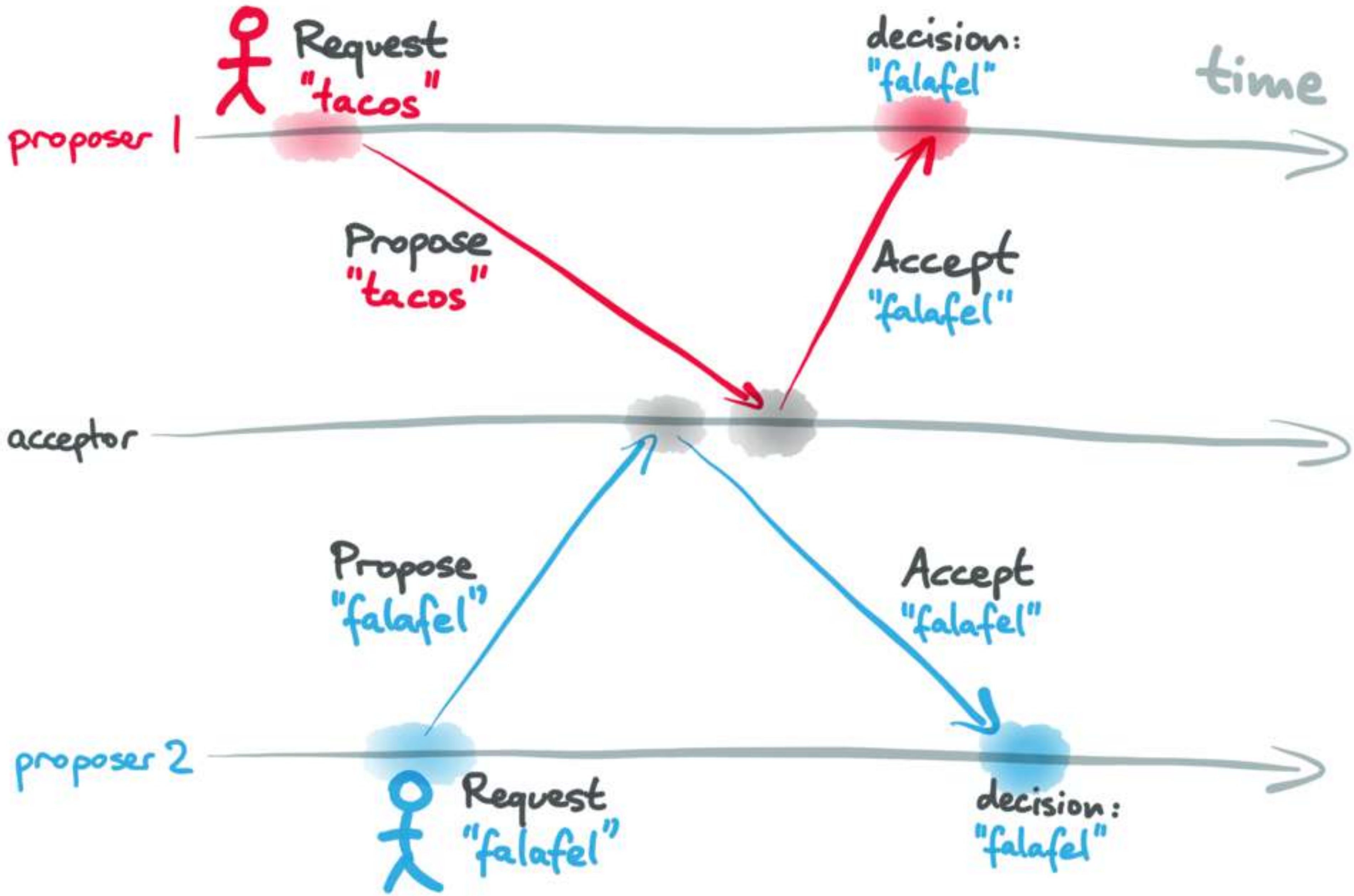

Request
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decision:
"falafel"
time
proposer 1
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Accept
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decision:
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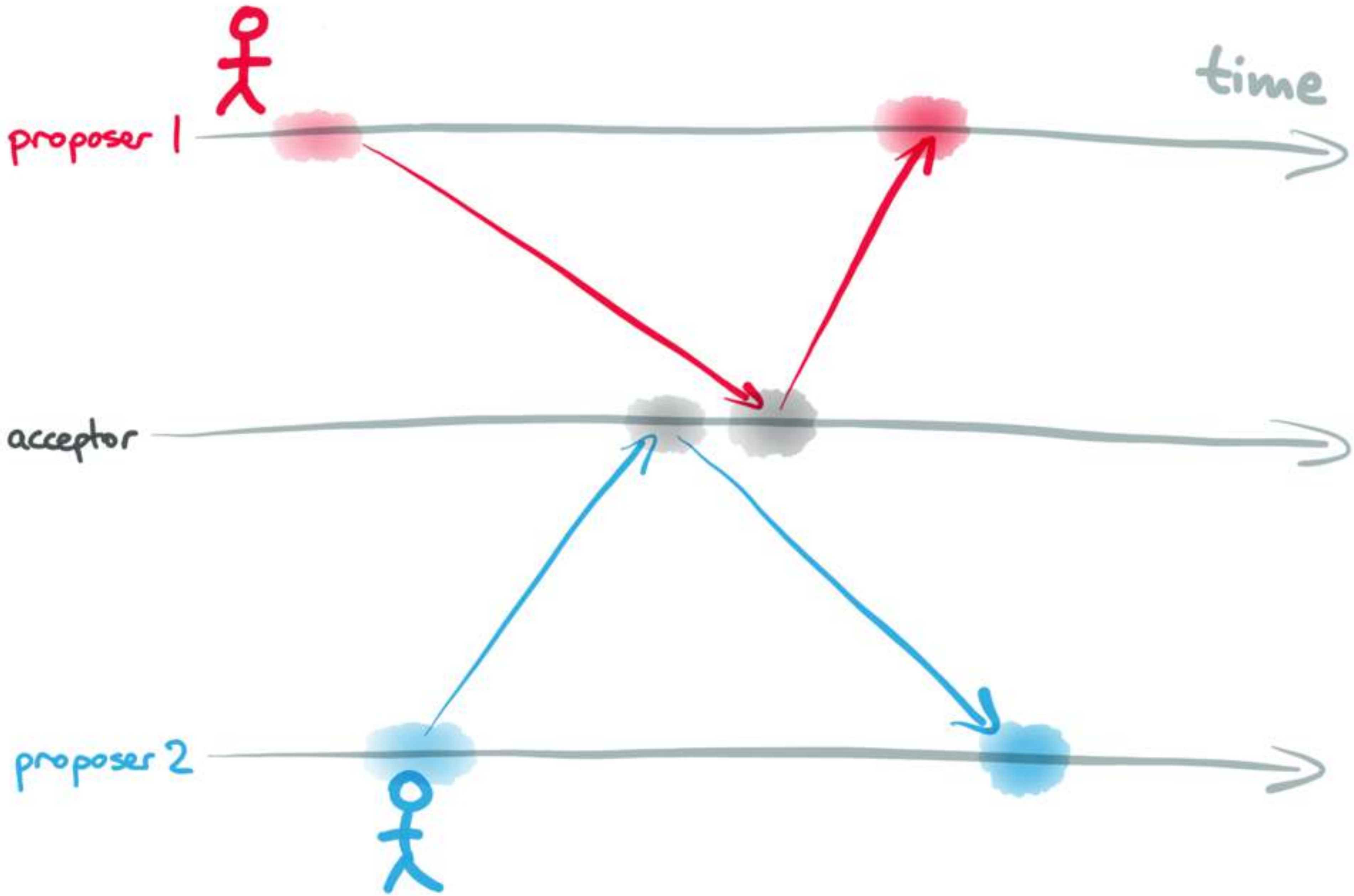
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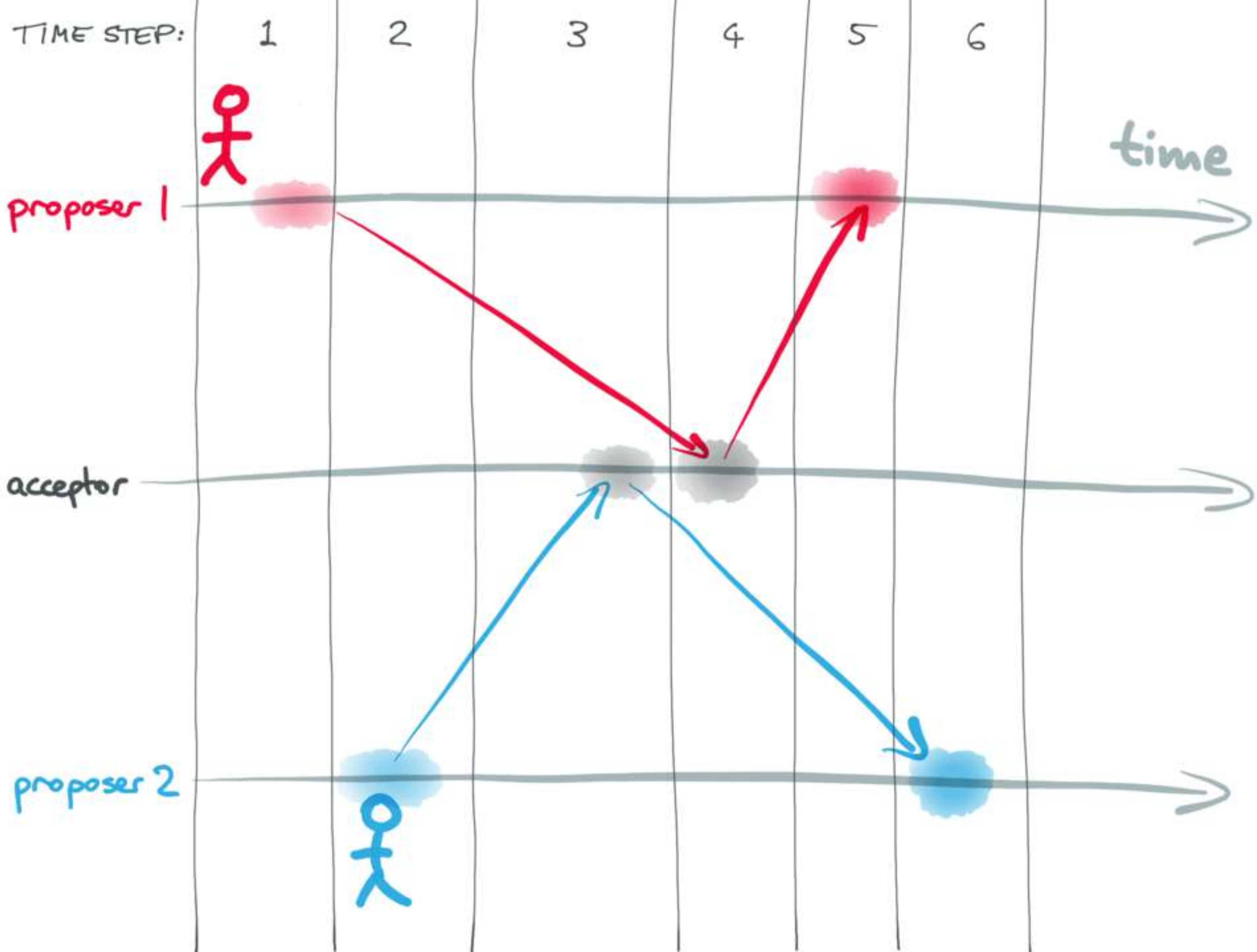
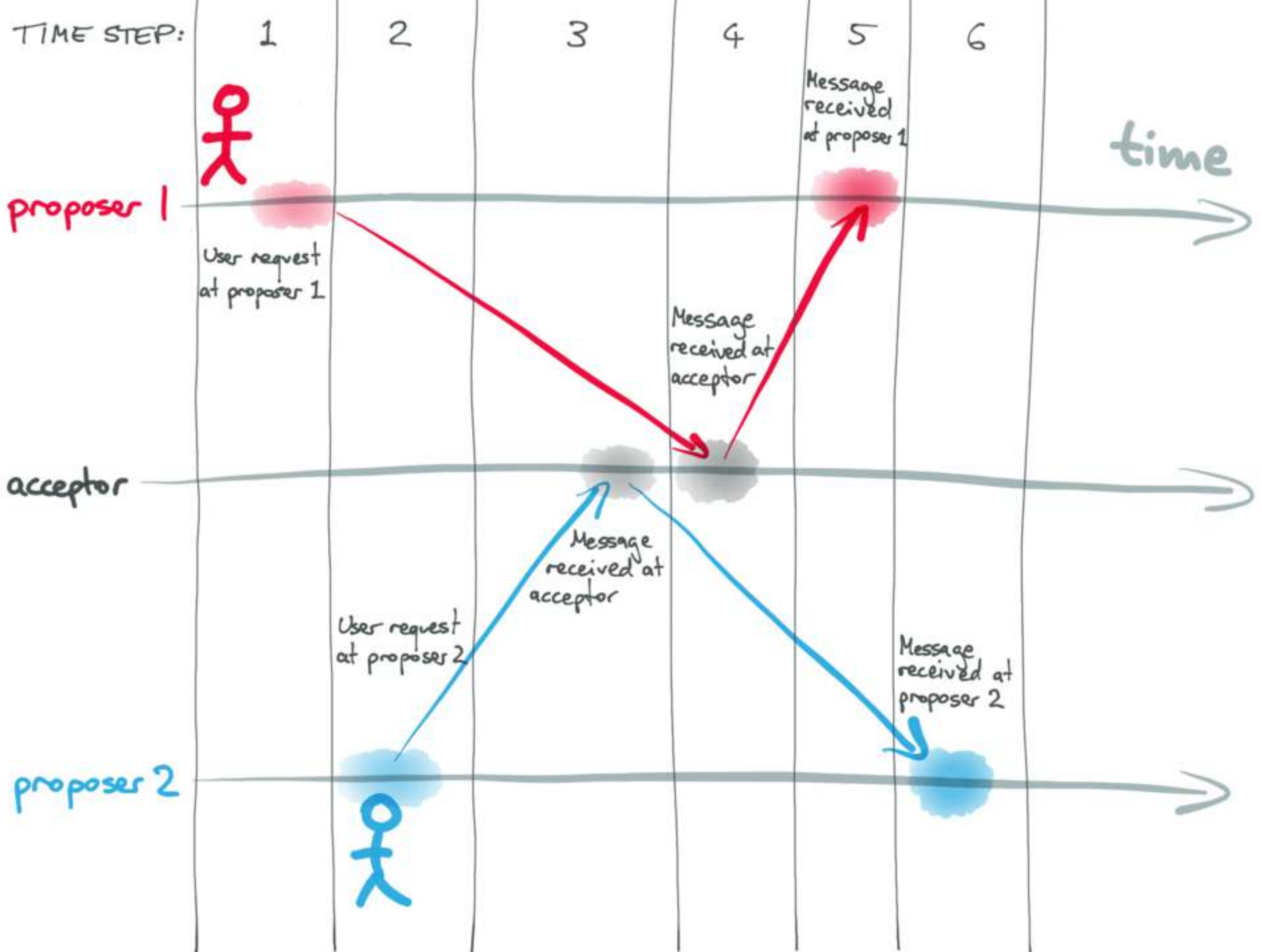

TIME STEP:
1
2
3
4
5
6
time
proposer 1
acceptor
proposer 2



TIME STEP:
1
2
3
4
5
6
time
proposer 1
acceptor
proposer 2
User request at proposer 1
Message received at acceptor
Message received at proposer 1
User request at proposer 2
Message received at acceptor
Message received at proposer 2

SYSTEM MODEL IN ISABELLE

- Linear sequence of time steps
  (like TLA+)

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- Linear sequence of time steps
  (Like TLA+)
- Each step: one process handles event
  (event types: user request, message received, timeout)
- Step function type signature:

$$\underbrace{\text{processID}}_{\text{who is executing?}} \Rightarrow \underbrace{\text{state}}_{\text{current local state}} \Rightarrow \underbrace{\text{event}}_{\text{what happened?}} \Rightarrow (\underbrace{\text{state}}_{\text{new local state}} \times \underbrace{\text{msg set}}_{\text{messages to send}})$$

PYTHON	ISABELLE/HOL

`def identity(x):`
`    return x`
	`fun identity where`
	`  ‹identity x = x›`
	OR
	`definition identity where`
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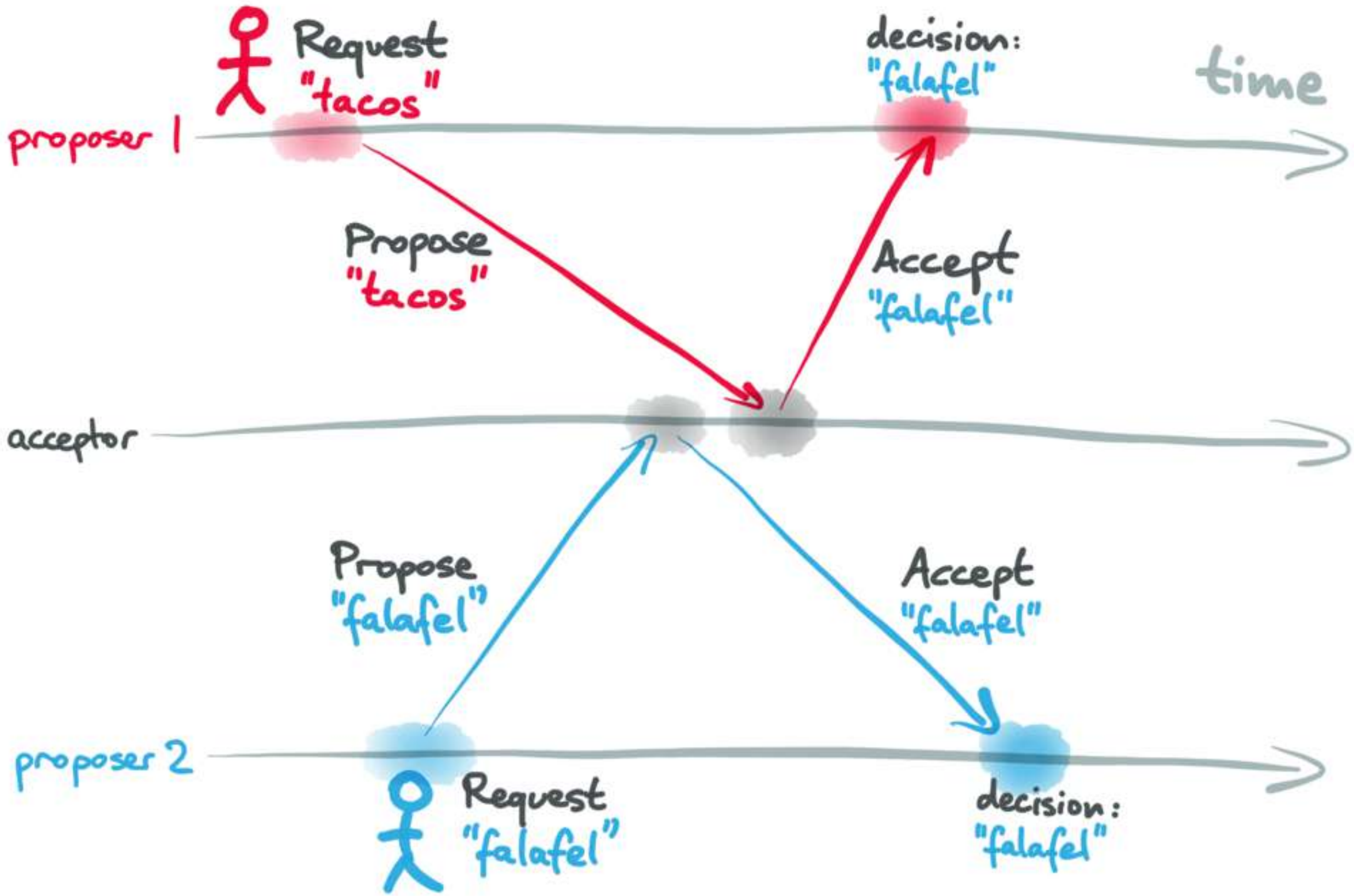
`lambda x: x`	`λx. x`
`identity(3)`	`identity 3`



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STEP FUNCTIONS

PROPOSER:

ON user request:
  send proposed value to ACCEPTOR
ON response from ACCEPTOR:
  learn decided value

ACCEPTOR:

ON proposal received from PROPOSER:
  IF value has been previously decided:
    send value to PROPOSER
  ELSE:
    decide proposed value
    send it to PROPOSER

PROOF ESSENTIALS

Logical implication:

$$\underbrace{P_1 \wedge P_2 \wedge \ldots \wedge P_n}_{\text{assumptions}} \Rightarrow \underbrace{Q}_{\text{consequent}}$$

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... also written as:

$$P_1 \Longrightarrow P_2 \Longrightarrow \ldots \Longrightarrow P_n \Longrightarrow Q$$

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THEOREM.

Assuming states are the states of all processes after executing any number of steps of the consensus algorithm

and states proc1 = Some val1
and states proc2 = Some val2

for any proc1, proc2

then we prove that val1 = val2.

INVARIANT 1:

For any proposer p, if p's state is Some val, then there exists a process a that has sent a message Accept val to p.

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INVARIANT 2:

If a message Accept val has been sent, then the acceptor is in the state Some val.

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$\forall x.\ P(x)$

for all values of x, the statement $P(x)$ is true

PROOF ESSENTIALS

$\forall x.\ P(x)$ — for all values of x, the statement $P(x)$ is true

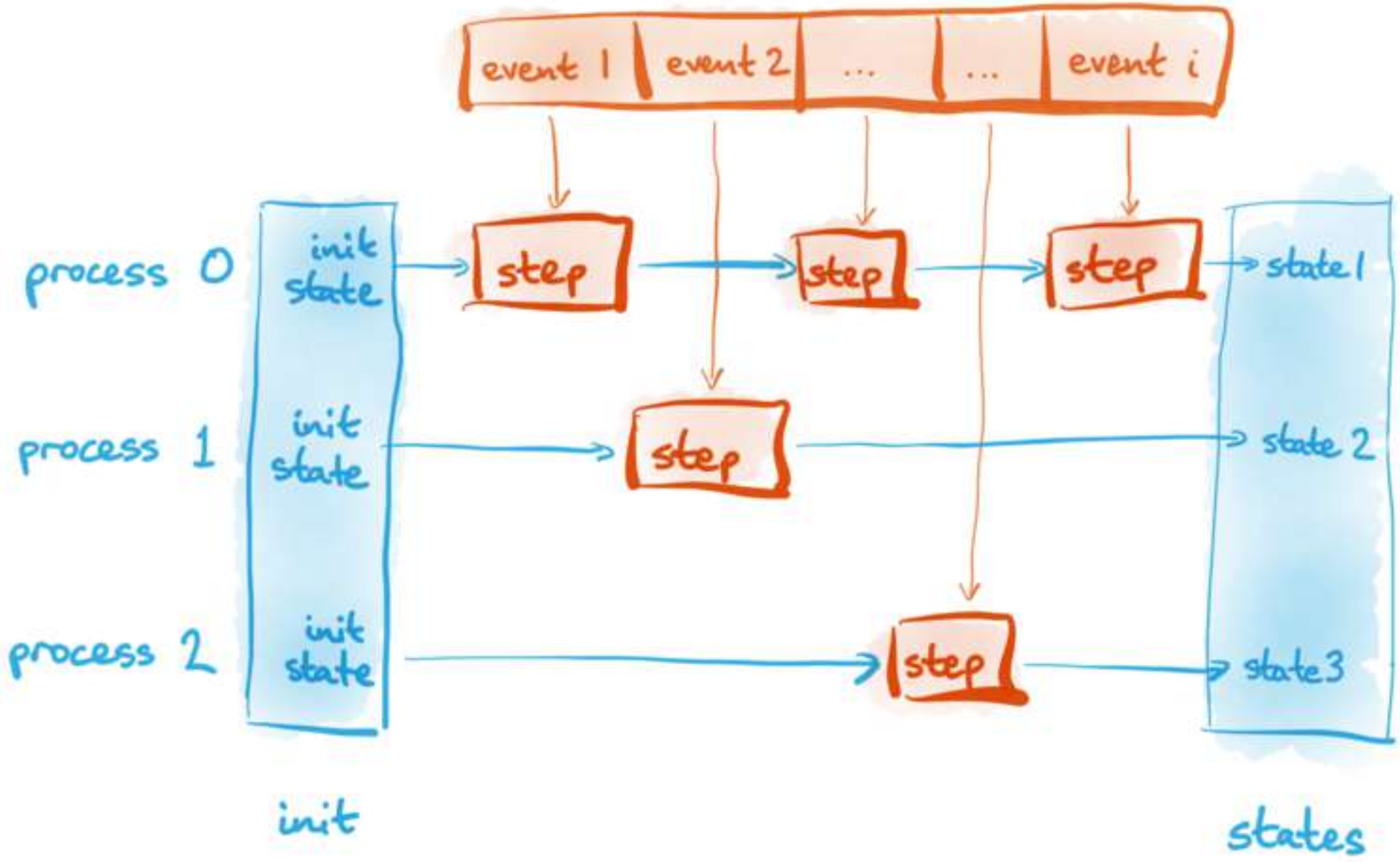
$\exists x.\ P(x)$ — there exists some value x for which the statement $P(x)$ is true

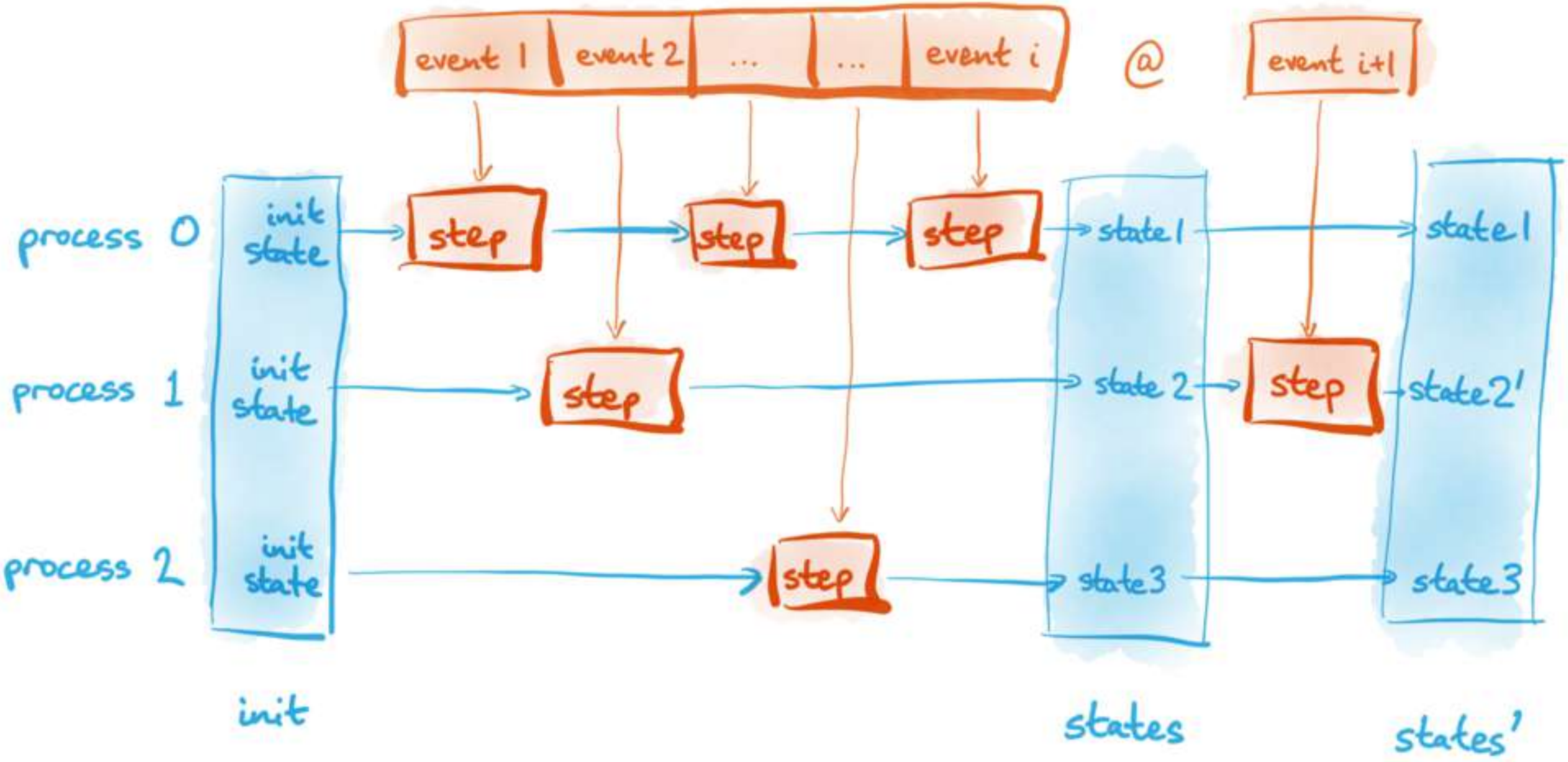
INVARIANT 1:

For any proposer p, if p's state is Some val, then there exists a process a that has sent a message Accept val to p.

INVARIANT 2:

If a message Accept val has been sent, then the acceptor is in the state Some val.


event 1
event 2
...
...
event i
process 0
process 1
process 2
init state
init state
init state
step
step
step
step
step
state 1
state 2
state 3
init
states



event 1
event 2
...
...
event i
@
event i+1
process 0
process 1
process 2
init state
init state
init state
step
step
step
step
step
step
state 1
state 2
state 3
state 1
state 2'
state 3
init
states
states'

PROOF TECHNIQUE

Induction on lists!

If we have $P([\,])$

and also $P(xs) \Rightarrow P(xs @ [x])$

then $P(xs)$ for all lists xs

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Induction on lists!

BASE CASE

If we have $P([\,])$

INDUCTIVE STEP

and also $P(xs) \Rightarrow P(xs @ [x])$

then $P(xs)$ for all lists xs

PROOF TECHNIQUE

Induction on lists!

BASE CASE

If we have $P([])$

INDUCTIVE STEP

and also $P(xs) \Rightarrow P(xs @ [x])$

then $P(xs)$ for all lists xs

Finite amount of proof effort, even though set of lists is infinite!

Full proof at:

https://martinkl.com/agree

Thanks!

Martin Kleppmann

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